

MATH40007 - An Introduction to Applied Mathematics

Revised Practice Paper 1

2026 structure reinforcement

Time allowed: 2 hours
 Total marks: 80
 Questions: Answer all four questions
 Historical role: One controlled paper close to 2026; not a claim of seven-year coverage

Instructions. Show all essential working. Use the methods taught in the course. Credit may be awarded for a correct method even when a later arithmetic error occurs. This is unofficial practice material, not an Imperial College examination paper.

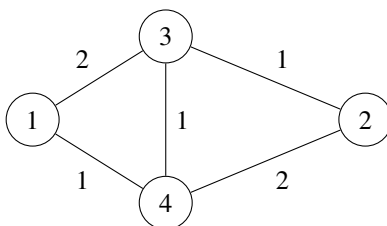
Question 1

(a) Consider

$$M = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \\ 4 & 8 & 4 \end{pmatrix}.$$

Find the rank of M and give bases for its column space, right null space, row space and left null space. (8 marks)

(b) In the circuit below, node 1 is held at voltage 1 and node 2 is grounded. Edge labels are conductances and Kirchhoff's current law holds at nodes 3 and 4.

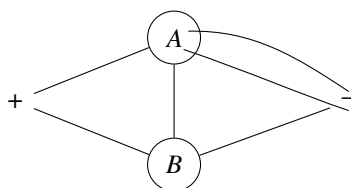


Find the voltages at nodes 3 and 4, the current in edge 3–4 including its direction, and the effective conductance between nodes 1 and 2. (12 marks) (Total: 20 marks)

Question 2

All edges in the diagrams below have unit conductance. Parallel edges are distinct edges.

- (a) The first network has terminals $+$ and $-$ and internal nodes A, B . The edges are $+A$, $+B$, two parallel edges $A-$, one edge $B-$ and one edge AB .



Find its effective conductance.

(8 marks)

- (b) A second network consists of two parallel paths from $+$ to $-$, one of length 2 and one of length 4. Find its effective conductance.

(6 marks)

- (c) Put the two networks in series, separated by one extra unit edge. A random walker starts at the first $+$, chooses uniformly among incident edges and stops on reaching the final $-$ or returning to its starting point. Find the probability of reaching $-$ first.

(6 marks)

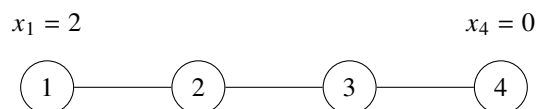
(Total: 20 marks)

Question 3

Diffusion takes place on the line graph 1 – 2 – 3 – 4. Boundary concentrations are fixed at

$$x_1(t) = 2, \quad x_4(t) = 0,$$

there are no sources at nodes 2 and 3, and $x_2(0) = 0, x_3(0) = 1$.



Let $\hat{x} = (x_2, x_3)^T$ and $K_2 = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$.

- (a) Derive the reduced differential equation, find the steady state and give an orthonormal eigensystem of K_2 . (8 marks)
- (b) Find explicit formulae for $x_2(t)$ and $x_3(t)$. (8 marks)
- (c) State the initial and limiting values and describe a correct sketch. (4 marks) (Total: 20 marks)

Question 4

A uniform rod occupies $0 \leq x \leq 1$. Its electrical conductivity is $\sigma = 2$ and thermal conductivity is $k = 3$.

(a) The endpoints are held at $\phi(0) = 0$, $\phi(1) = 1$, with no interior current sources. Find ϕ , the electrical current density $J = -\sigma\phi'$ and the electrical dissipation density $q_e = -\phi'J$. The ends are then held at temperatures $T(0) = 0$, $T(1) = 1$, and all electrical dissipation becomes heat. Find the steady temperature. (10 marks)

(b) Replace the thermal condition at $x = 0$ by insulation, retaining $T(1) = 1$. Find the new temperature and the outward heat flux at $x = 1$. Verify the answer by an energy balance. (10 marks) (Total: 20 marks)