

MATH40007 - An Introduction to Applied Mathematics

Revised Practice Paper 6

Seven-year mixed transfer pressure test

Time allowed: 2 hours
 Total marks: 80
 Questions: Answer all four questions
 Historical role: Topics reordered to prevent recognition by 2026 question position

Instructions. Show all essential working. Use the methods taught in the course. Credit may be awarded for a correct method even when a later arithmetic error occurs. This is unofficial practice material, not an Imperial College examination paper.

Question 1

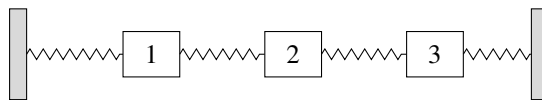
A unit conductor occupies the first quadrant. Let $z_0 = 1 + i$ and

$$h(z) = -\frac{m}{2\pi} \log \left(\frac{z^2 - z_0^2}{z^2 - \bar{z}_0^2} \right), \quad \phi = \Re h(z).$$

- (a) Show that both coordinate axes are grounded. (4 marks)
- (b) Identify the source inside the conductor. (4 marks)
- (c) Find the complex current density. (4 marks)
- (d) Find the outward normal-current density on each grounded axis. (4 marks)
- (e) Calculate the flux through each axis and verify current conservation. (4 marks) (Total: 20 marks)

Question 2

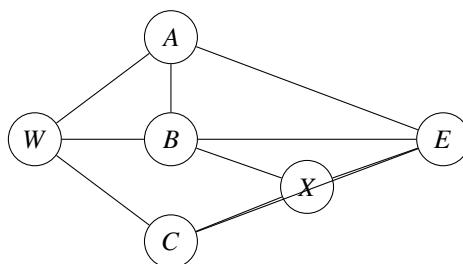
Three unit masses move between two fixed walls. Unit springs join the left wall to mass 1, successive masses, and mass 3 to the right wall.



- (a) Write the stiffness matrix and find an orthonormal eigensystem and the natural frequencies. (8 marks)
- (b) A constant force $f = (1, 0, 1)^T$ is applied. Find the equilibrium. (4 marks)
- (c) The force is removed at $t = 0$ with initial displacement equal to the equilibrium and initial velocity $(0, 1, 0)^T$. Find the free motion in modal form. (8 marks) *(Total: 20 marks)*

Question 3

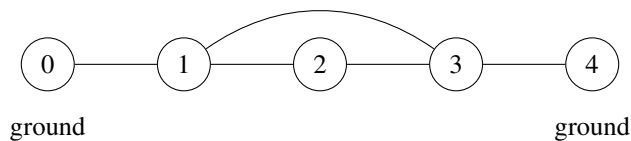
Consider the graph below, with terminals W and E .



- (a) Give the dimensions and nullities of an incidence matrix. (4 marks)
- (b) A walker starts at W , leaves it and stops on reaching E or returning to W . Find the probability of reaching E first. (8 marks)
- (c) If W is held at voltage 1 and E grounded, find the voltage at X . (8 marks) (Total: 20 marks)

Question 4

Nodes 0 and 4 are grounded. Internal nodes 1,2,3 are connected by the path $0 - 1 - 2 - 3 - 4$ and there is one extra unit edge $1 - 3$.



- (a) Write the reduced Laplacian K . (4 marks)
 - (b) Find the Green matrix $M = K^{-1}$. (8 marks)
 - (c) A unit current is injected at node 2 and removed through the grounds. Find the internal voltages and verify that the total ground current is one. (4 marks)
 - (d) A unit current is injected at node 1 and removed at node 3. Find the effective resistance and conductance between nodes 1 and 3. (4 marks)
- (Total: 20 marks)*