

Paper 1 - Solutions and Marking Guide

2026 structure reinforcement

Question 1

Recognition cue. The columns are visibly dependent, so determine one pivot column first and then use rank-nullity.

(a) The columns are $c_2 = 2c_1$ and $c_3 = c_1$, so $\text{rank } M = 1$.

$$\text{Col } M = \text{span}\{(1, 2, 3, 4)^T\}, \quad \text{Row } M = \text{span}\{(1, 2, 1)\}.$$

The equation $x_1 + 2x_2 + x_3 = 0$ gives

$$\mathcal{N}(M) = \text{span}\{(-2, 1, 0)^T, (-1, 0, 1)^T\}.$$

The left-null condition $y_1 + 2y_2 + 3y_3 + 4y_4 = 0$ gives, for example,

$$\mathcal{N}(M^T) = \text{span}\{(-2, 1, 0, 0)^T, (-3, 0, 1, 0)^T, (-4, 0, 0, 1)^T\}.$$

(b) Put $v_3 = a$, $v_4 = b$. KCL gives

$$2(a - 1) + a + (a - b) = 0, \quad (b - 1) + 2b + (b - a) = 0.$$

Hence $a = 3/5$, $b = 2/5$. The edge current is $a - b = 1/5$, from 3 to 4. The input current is

$$2(1 - a) + (1 - b) = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}.$$

so $C_{\text{eff}} = 7/5$. **Checks.** Potentials lie between 0 and 1; current flows from the higher-potential node; input and output currents balance.

Question difficulty allocation: $A = 8$, $B = 12$, $C = 0$, $D = 0$.

Question 2

Recognition cue. Solve harmonic equations for the bridge, then use series/parallel resistance and the escape-probability identity $p = C_{\text{eff}}/\deg(+)$.

(a) Let the potentials at A, B be a, b . Then

$$4a - b = 1, \quad -a + 3b = 1,$$

so $a = 4/11$, $b = 5/11$. The input current is

$$(1 - a) + (1 - b) = \frac{13}{11}.$$

(b) The two path conductances are $1/2$ and $1/4$, hence $C_B = 3/4$.

(c) The total resistance is

$$R = \frac{11}{13} + 1 + \frac{4}{3} = \frac{124}{39}, \quad C = \frac{39}{124}.$$

The initial terminal has degree 2, so

$$p = \frac{C}{2} = \frac{39}{248}.$$

Checks. All equivalent conductances are positive and smaller than the total number of edges leaving the source.

Question difficulty allocation: $A = 6$, $B = 0$, $C = 8$, $D = 6$.

Question 3

Recognition cue. For a forced diffusion system, first find the steady state, then expand the initial deviation rather than the steady state.

The reduced equation is

$$\hat{x}' = -K_2 \hat{x} + \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

The steady state is $x_s = (4/3, 2/3)^T$. An orthonormal eigensystem is

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \lambda_1 = 1; \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \lambda_2 = 3.$$

Since $\hat{x}(0) - x_s = (-4/3, 1/3)^T$, the modal coefficients are

$$\alpha_1 = -\frac{1}{\sqrt{2}}, \quad \alpha_2 = -\frac{5}{3\sqrt{2}}.$$

Thus

$$x_2(t) = \frac{4}{3} - \frac{1}{2}e^{-t} - \frac{5}{6}e^{-3t}, \quad x_3(t) = \frac{2}{3} - \frac{1}{2}e^{-t} + \frac{5}{6}e^{-3t}.$$

The initial values are 0, 1 and the limits are $4/3, 2/3$. The first component rises monotonically; the second initially falls and later rises to its limit. **Checks.** Substitute $t = 0$ and $t \rightarrow \infty$ immediately. The exponents must be the eigenvalues 1 and 3.

Question difficulty allocation: $A = 8, B = 8, C = 4, D = 0$.

Question 4

Recognition cue. Use current conservation before solving the heat equation.

No source implies $J' = 0$, so $\phi'' = 0$. Therefore

$$\phi = x, \quad J = -2, \quad q_e = -\phi' J = 2.$$

The heat equation is $-3T'' = 2$. With $T(0) = 0, T(1) = 1$,

$$T = -\frac{x^2}{3} + \frac{4x}{3}.$$

For insulation at $x = 0, T'(0) = 0$ and $T(1) = 1$, so

$$T = \frac{4}{3} - \frac{x^2}{3}.$$

The outward flux at $x = 1$ is $-3T'(1) = 2$. This equals $\int_0^1 q_e dx = 2$. **Question difficulty allocation:** $A = 10, B = 0, C = 0, D = 10$.