

Paper 6 - Solutions and Marking Guide

Seven-year mixed transfer pressure test

Question 1

On either coordinate axis, z^2 is real and the numerator and denominator are complex conjugates, so their ratio has modulus 1 and $\phi = 0$. Inside the first quadrant the only finite logarithmic singularity is at $z_0 = 1 + i$, a source of strength m . Differentiation gives

$$J_x - iJ_y = \frac{mz}{\pi} \left(\frac{1}{z^2 - z_0^2} - \frac{1}{z^2 - \bar{z}_0^2} \right).$$

On the positive x -axis,

$$J_x = 0, \quad J_y = -\frac{4mx}{\pi(x^4 + 4)},$$

so the outward density is $4mx/[\pi(x^4 + 4)]$. On the positive y -axis,

$$J_x = -\frac{4my}{\pi(y^4 + 4)}, \quad J_y = 0,$$

and the outward density is $4my/[\pi(y^4 + 4)]$. Each flux is

$$\int_0^\infty \frac{4ms}{\pi(s^4 + 4)} ds = \frac{m}{2},$$

so the total outward flux is m , matching the source strength. **Question difficulty allocation:** $A = 8, B = 4, C = 4, D = 4$.

Question 2

The stiffness matrix is

$$K = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

An orthonormal eigensystem is

$$\phi_1 = (1/2, 1/\sqrt{2}, 1/2)^T, \quad \lambda_1 = 2 - \sqrt{2},$$

$$\phi_2 = (1/\sqrt{2}, 0, -1/\sqrt{2})^T, \quad \lambda_2 = 2,$$

$$\phi_3 = (1/2, -1/\sqrt{2}, 1/2)^T, \quad \lambda_3 = 2 + \sqrt{2}.$$

The frequencies are $\omega_j = \sqrt{\lambda_j}$. Since $K(1, 1, 1)^T = (1, 0, 1)^T$, the equilibrium is $x_e = (1, 1, 1)^T$. Let $a_j = \phi_j^T x_e$ and $b_j = \phi_j^T(0, 1, 0)^T$. Then

$$a_1 = 1 + \frac{1}{\sqrt{2}}, \quad a_2 = 0, \quad a_3 = 1 - \frac{1}{\sqrt{2}}; \quad b_1 = \frac{1}{\sqrt{2}}, \quad b_2 = 0, \quad b_3 = -\frac{1}{\sqrt{2}}.$$

Therefore

$$x(t) = \sum_{j=1}^3 \left(a_j \cos \omega_j t + \frac{b_j}{\omega_j} \sin \omega_j t \right) \phi_j.$$

Question difficulty allocation: $A = 8, B = 6, C = 2, D = 4$.

Question 3

There are 10 edges and 6 vertices. The incidence matrix is 10×6 , rank 5, right-null dimension 1 and left-null dimension 5. With boundary values $E = 1, W = 0$, harmonic equations give

$$V_A = \frac{41}{79}, \quad V_B = \frac{44}{79}, \quad V_C = \frac{45}{79}, \quad V_X = \frac{56}{79}.$$

After leaving W , the walker chooses A, B, C equally, hence

$$P(E \text{ before return } W) = \frac{41 + 44 + 45}{3 \cdot 79} = \frac{130}{237}.$$

With $W = 1, E = 0$, the voltage is the complement, so

$$V_X = 1 - \frac{56}{79} = \frac{23}{79}.$$

Question difficulty allocation: $A = 4, B = 6, C = 6, D = 4$.

Question 4

The reduced Laplacian is

$$K = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 3 \end{pmatrix}.$$

Direct inversion gives

$$M = K^{-1} = \begin{pmatrix} 5/8 & 1/2 & 3/8 \\ 1/2 & 1 & 1/2 \\ 3/8 & 1/2 & 5/8 \end{pmatrix}.$$

For $f = e_2$,

$$x = M e_2 = (1/2, 1, 1/2)^T.$$

The ground currents through $0 - 1$ and $3 - 4$ are $1/2$ and $1/2$, totalling one. For $f = e_1 - e_3$,

$$x = M f = (1/4, 0, -1/4)^T.$$

The voltage difference is $1/2$, so

$$R_{13} = \frac{1}{2}, \quad C_{13} = 2.$$

Question difficulty allocation: $A = 12, B = 4, C = 0, D = 4$.

Pack-level difficulty audit

Every paper has 80 marks and the same aggregate official-style distribution as the 2026 paper:

$$A = 32, \quad B = 20, \quad C = 12, \quad D = 16.$$

The content and question order vary across historical ability families; the equality is at paper-level difficulty, not at fixed 2026 topic slots.